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END-POINT BEHAVIOR OF EXCLUSIVE PROCESSES: THE TWILIGHT REGIME OF PERTURBATIVE QCD *

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A selected set of topics along the borderline between perturbative and nonperturbative QCD in exclusive reactions are studied. Specific problems, related to different mechanisms of momentum transfer to an intact hadron, are discussed. Calculations of the space-like form factors of the pion and the nucleon are reviewed within a convolution scheme of short-distance (hard) and large-distance (soft) contributions which takes into account soft gluon emission and the intrinsic transverse hadron size. The failure of this scheme to reproduce the existing experimental data signals sizeable higher-order perturbative corrections (a K-factor of order two) and/or higher-twist contributions.

1 Introduction

This article presents an overwiew of exclusive processes, focusing on their end-point behavior. To set the stage, we discuss and review problems relating to the (momentum) scales involved in form factor calculations: scale locality, infrared (IR) safety, gluonic radiative corrections, and the role of hadronic size effects. These issues are more precisely described in terms of the essential mechanisms of momentum transfer to an intact hadron. We then use detailed calculations to investigate how these effects influence the predictions for $F_\pi(Q^2)$, $G_M^p(Q^2)$, and $G_M^n(Q^2)$ relative to existing data.

The application of perturbative Quantum ChromoDynamics (pQCD) to inclusive processes has been very successful and predictive and there is now ample experimental verification for a variety of reactions. In contrast, exclusive processes, though of basic importance for a deeper understanding of confinement, are yet not so rigorously established. In order that pQCD becomes applicable at the *amplitude level*, a short-distance part of the strong-interaction amplitude has to be isolated. This is then amenable to perturbative analysis within a hard-scattering scheme. Beyond this, however, one has to use additional (unsettled) nonperturbative methods to model the hadron wave functions which encode bound-state features. Since hadron wave functions appear in *integrated* quantities they are not directly accessible to experiment.

Once factorization of regimes has been accomplished, renormalization group (RG) techniques can be employed to calculate the evolution behavior of the factorized parts. The logarithmic scaling violations are found to be controlled by the same nonsinglet anomalous dimensions as in deep-inelastic scattering [1,2].

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2 Factorization in exclusive processes

Factorization theorems are of central importance in quantum field theory. The basic idea is that one can separate high-momentum from low-momentum dependence in a multiplicative way. For example, proving that ultraviolet (UV) divergences occuring in Feynman graphs can be absorbed into multiplicative renormalization factors (infinite constants) is instrumental in establishing renormalizability of the theory. The technical difficulty is to prove factorization of a particular QCD process to *all-orders* in the coupling constant going beyond leading logarithms [3]. These difficulties derive from the fact that in QCD a new type of IR-divergence is encountered, the *collinear* divergence, and that in higher orders the self-coupling of gluons becomes important in the exponentiation of IR-divergences.

The realization of factorization when applying to elastic form factors can be written in the form of a convolution of a hard-scattering amplitude (dubbed T_H) describing the short-distance quark-gluon interactions, and two soft wave functions corresponding to the incoming and outcoming hadron [1,4]. Generically,

$$F(Q^2) = \Phi^{out}(m/\mu) \otimes T_H(\mu/Q) \otimes \Phi^{in}(m/\mu) \; , \tag{1}$$

where m sets the typical virtuality in the soft parts and Q is the (external) scale characteristic of the hard (parton) subprocesses. The matching scale μ at which factorization has been performed is arbitrary and, assuming that $\mu \gg m$, it can be safely identified with the renormalization scale – unavoidable in any perturbative calculation – by virtue of the RG equations. In this way, F can be rewritten as a function only of the coupling constant operative at that scale.

As long as *scale locality* is preserved, i.e., the variation of the effective coupling constant with μ is governed by the same momentum scale, and the limit $m \to 0$ is finite, Eq. (1) is valid because *intrusions* from the hard into the soft regime are prohibited. This means that T_H is insensitive to long-distance interactions, i.e., it is *IR safe*. All IR-sensitivity resides in the hadron distribution amplitudes $\Phi^{in(out)}$ which are independent of large momentum scales and may depend on the external scale Q^2 only through RG-evolution. Both the subtraction procedure of UV poles in the soft parts and the cancellation of IR divergences in the hard part are not uniquely fixed. Nevertheless, they have to ensure that the asymptotic behavior of F is IR-insensitive and governed by the leading anomalous dimensions associated with vertex and quark self-energy corrections.

Adopting a factorization scheme, the initial (final) state of the hadron has a certain probability distribution for finding its valence quarks carrying longitudinal momentum fractions $0 \leq x_i = k_i^+/P^+ \leq 1$ in a $P^3 \to \infty$ frame. Apart from the slow perturbative Q^2-evolution, this randomness depends only on the uncalculable confinement dynamics and not on the specific hard-scattering collision, i.e., it is *universal*. Hence

$$\Phi(x_i, \mu^2) \equiv \left(\ln \frac{\mu^2}{\Lambda_{QCD}^2} \right)^{-c\,\gamma_F/\beta} \int_0^{\mu^2} \prod_{i=1}^{N} [d^2 \vec{k}_\perp^i] \psi(x_i, \vec{k}_\perp^{(i)}) \; , \tag{2}$$

where $N=2$, $c=1$ for the meson and $N=3$, $c=2/3$ for the nucleon, respectively, and γ_F is the anomalous dimension associated with quark self-energy.

To solve the evolution equation, $\Phi^{(H)}$ for hadron H has to be expressed as an orthogonal expansion in terms of appropriate hypergeometric functions which constitute an eigenfunction basis of the gluon-exchange potential, i.e.,

$$\Phi^{(H)}(x_i,Q^2)=\Phi_{as}^{(H)}(x_i)\sum_{n=0}^{\infty}B_n^{(H)}(\mu^2)\tilde{\Phi}_n^{(H)}(x_i)\exp\left\{\int_{\mu^2}^{Q^2}\frac{d\bar{\mu}}{\bar{\mu}}\gamma_F(g(\bar{\mu}^2))\right\}, \tag{3}$$

where $\Phi_{as}^{(H)}$ is the asymptotic amplitude (see below) over fractional momenta proportional to the weight $w(x_i)$ of the orthogonal basis, and $\tilde{\Phi}_n^{(H)}$ denote the corresponding eigenfunctions. The coefficients $B_n^{(H)}$ of this expansion are associated with matrix elements of composite lowest-twist operators with definite anomalous dimensions taken between the vacuum and the external hadron. In the meson case (leading twist=2), the eigenfunctions of the diagonalized evolution equation [1] are the Gegenbauer polynomials $C_n^{3/2}$ [5], which correspond to conformal operators [6] with associated anomalous dimensions given by

$$\gamma_n^{(\pi)}=\frac{C_F}{\beta}\left[1+4\sum_{n=0}^{n+1}\frac{1}{k}-\frac{2}{(n+1)(n+2)}\right]\geq 0 \qquad (n \text{ even}) . \tag{4}$$

Introducing the relative coordinate $\xi=x_1-x_2$, orthogonality with respect to the weight $w(\xi)=(1-\xi^2)$ yields expansion coefficients going like

$$B_n^{(\pi)}\left(\ln\frac{Q^2}{\Lambda_{QCD}^2}\right)^{-\gamma_n}=\frac{2(2n+3)}{(2+n)(1+n)}\int_{-1}^{1}d\xi\, C_n^{3/2}(\xi)\ \Phi^{(\pi)}(\xi,Q^2), \tag{5}$$

meaning that for higher orders they decrease like $1/n^2$, provided $\Phi^{(\pi)}(x_i,\mu)\leq Kx_i^{\epsilon}$ as $x_i\longrightarrow 0$ for some $\epsilon>0$ [1].

In the nucleon case (leading twist=3), an orthogonal normalized basis of the evolution kernel is provided by linear combinations of Appell polynomials which depend on two variables [1,7]. Here orthogonality alone is insufficient to fix the eigenfunctions uniquely. It was first shown in [8] that the expansion coefficients B_n become analytically tractable up to any desired polynomial order $M\geq i+j$ in terms of *strict* moments

$$\Phi_N^{(i0j)}(\mu^2)=\int_0^1[dx]\,x_1^i\,x_2^0\,x_3^j\,\Phi_N(x_k,\mu^2) \tag{6}$$

of the mixed-symmetry nucleon distribution amplitude [9] Φ_N:

$$\frac{B_n^{(N)}(Q^2)}{\sqrt{N_n}}=\frac{\sqrt{N_n}}{120}\,B_n^{(N)}(\mu^2)\left[\frac{\ln(Q^2/\Lambda_{QCD}^2)}{\ln(\mu^2/\Lambda_{QCD}^2)}\right]^{-\gamma_n}\sum_{i,j=0}^{\infty}a_{ij}^n\ \Phi_N^{(i0j)}(\mu^2) . \tag{7}$$

The projection coefficients a^n_{ij}, the anomalous dimensions of trilinear twist-three quark operators

$$\gamma_n^{(N)} = \frac{1}{\beta}\left(\frac{3}{2}C_F + 2\eta_n C_B\right) \tag{8}$$

(where η_n are the zeros of the characteristic polynomial that diagonalizes the evolution equation), and the normalization constants N_n up to order $M = 4$ are tabulated in [7].

The moment values – calculated, for instance, via QCD sum rules [9,10] – provide only *local* constraints that are not sufficient for the distribution amplitude to be reconstructed in an unambigous way [11]. One can always add some oscillating function which vanishes at the points fixed by the local constraints but which contributes outside. Thus one has to impose *global* constraints as well, for shaping the distribution amplitude as a *whole*. Such constraints have been successfully used in Refs. [7] - [15] to ensure dominance of the lowest-order contributions and minimize the influence of disregarded higher-order terms. In [14] a complete set of nucleon distribution amplitudes was determined which satisfy existing QCD sum rules [10,16] with comparable degree of accuracy while avoiding unphysical oscillations. These solutions organize themselves across a "fiducial orbit" that is characterized by a scaling relation between the form-factor ratio $|G^n_N|/G^p_M$ and the expansion coefficient B_4 (cf. Eq. (7)).

Hadron distribution amplitudes derived this way from QCD sum rules show an asymmetric balance of longitudinal momentum fractions of valence quarks. Convoluted with the corresponding hard-scattering amplitudes they lead to form factors which have the right magnitude and QCD-evolution behavior [7]. On the other hand, it was pointed out in [17,18] that asymmetric distribution amplitudes enhance the contributions of end-point regions and that, extracting these regions, the leading perturbative contribution to the form factor is reduced to a small fraction. This depletion of the form factor indicates sensitivity to the gluon offshellness in the end-point region, presumed to be large. Hence the perturbative treatment turns out to be inconsistent, meaning that uncalculated higher-order terms may be important, even leading. To reinstate the validity of pQCD, end-point contributions have clearly to be suppressed. To this end, a modified convolution scheme [19] – still within the hard-scattering picture – will be discussed below which incorporates Sudakov suppression due to radiative gluon corrections.

3 Momentum transfer mechanisms

There are basically two schemes for describing the transfer of a large external momentum Q to an intact hadron during elastic scattering: hard-gluon exchange and the Feynman mechanism. We begin with the first one which is tightly connected to pQCD and relies on the factorization theorem. Following this rationale, the struck quark connects to the other valence quarks via highly off-shell gluon propagators, meaning that the transverse interquark distances are rather small, *viz.* of order $1/Q$ and that *all* partons share comparable fractions of longitudinal momentum. Thus T_H can be reliably computed as a power series in the running coupling constant α_s [1,6] (see the lhs of Fig. 1).

In the asymptotic limit $Q^2 \to \infty$, only the $n = 0$ term in Eq. (3) survives so that

the RG-asymptote of the distribution amplitude reads ($\gamma_0 < \gamma_n$ for all $n > 0$)

$$\Phi_{as}^{(H)}(Q^2 \to \infty) = B_0^{(H)} w(x_i) \lim_{Q^2 \to \infty} \left(\ln \frac{Q^2}{\Lambda_{QCD}} \right)^{-C/\beta} \tag{9}$$

with $C = C_F$ (pion) and $C = 3C_F/2 - 2C_B$ (nucleon), where $B_0^{(H)}$ is the hadron wave function at the origin of coordinate space and the limit of the logarithm amounts to the wave-function renormalization factor Z_2. From these distribution amplitudes one infers that asymptotically the most likely configurations are those in which the valence quarks share longitudinal momentum in a uniform way, i.e., $x_i = 1/2$ for the pion and $x_i = 1/3$ for the nucleon. Within this scheme, when confinement sets in, a quark is not able to venture too far from the antiquark (in the pion) or the other two valence quarks (in the nucleon). This poses constraints on the offshellness of the involved propagators, typified by $x_i x'_j > \left(\Lambda_{QCD}^2/Q^2\right) \exp\left(4\pi/\beta\alpha_s\right)$. The problem then is that in the end-point regions $x_i, x'_j \to 0, 1$ the gluons become nearly on-shell (i.e., real) and the hard-gluon exchange mechanism becomes unreliable. This is also reflected in the behavior of form factors. As a rule, narrow distribution amplitudes yield for reasonable values of α_s results which are unrealistically low to be consistent with the data. Obviously, *broad* distribution amplitudes are required to match the data.

One method [9] extracts distribution amplitude moments from QCD sum rules using *local* vacuum condensates. In the pion case, the large values obtained this way for the moments can only be realized by a "double-humped" distribution amplitude of the form $\phi_{CZ}^{(\pi)}(x) = 30 f_\pi x(1-x)(1-2x)$ which is end-point dominated, whereas the central region ($x_i = 1/2$) is depleted. This distribution amplitude yields a pion form factor in much better agreement with data but at the expense that it accumulates its main contributions from the end-point region where a perturbative treatment is less reliable.

The approach taken in [20] is conceptually quite different. To avoid the inherent deficiencies of moment inversion (see for criticism [18]), the pion distribution amplitude was computed directly from QCD sum rules via dispersion relations. This approach makes use of *nonlocal* vacuum condensates which afford for the finite average virtuality of the vacuum quarks. Parametrizing the nonlocal quark condensate by a Gaussian, a model distribution amplitude was obtained which gives lowest moments $< \xi^n >$ with $n = 2, 4, 6$ close to those of $\Phi_{as}^{(\pi)} = f_\pi 6x(1-x)$, but which has a significantly wider shape and no dip in the central region.

Concerning the nucleon the situation is technically more complicated, though, perhaps, less prone to criticism. Up to now, all derived model distribution amplitudes (see, e.g., [7,15]) rely on moment-inversion techniques within rather large uncertainty intervals. However, a consistent pattern has emerged [12,14] which seems to encapsulate the main characteristics of the true nucleon distribution amplitude.

Let us consider now the other basic mechanism for elastic scattering due to Feynman [21]. In this scheme, almost all of the hadron's momentum is carried off by a

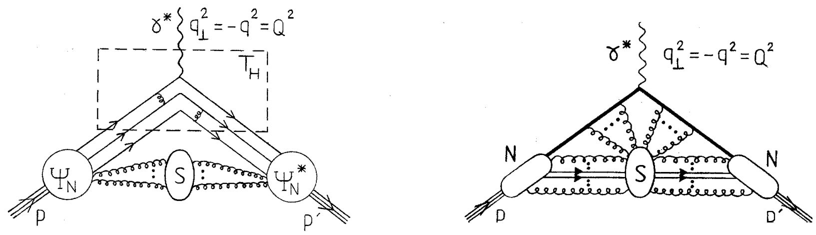

Figure 1: Mechanisms for momentum transfer during elastic scattering. The lhs shows hard-gluon exchange within pQCD. The blob S containing soft gluon lines (and an analogous one with soft quark-antiquark lines not shown here) spoils factorization but is power-suppressed, i.e., non-leading. The (rhs) shows the Feynman mechanism using for purposes of illustration quark and gluon lines. The leading quark is denoted by a heavy line, while all other lines represent wee quarks and soft gluons.

single parton, the others being "wee". This picture is consistent with a configuration in which only the struck quark is within an impact distance $1/Q$ of the electron while all other partons have rather random positions in the transverse direction, building a soft "cloud" with transverse size $\gg 1/Q$. Once the elastic scattering has happened, rearrangements are necessary to change quarks and gluons into hadrons. This conversion procedure (visualized on the rhs of Fig. 1) is controlled by the overlap of the initial and final state wave functions and cannot be computed within pQCD.

For the remainder of this report we will consider only calculations which are based on the hard-scattering picture.

4 Modified Convolution Scheme

In deriving Eq. (1), we tacitly assumed that the $k_\perp$-dependence of the quark and gluon propagators in T_H can be ignored. This is tantamount to factorizing the $k_\perp$-dependence into the distribution amplitudes which are the wave functions integrated over $k_\perp$ up to the factorization scale. Then, in the limit $Q^2 \to \infty$, the only gluon radiative corrections remaining uncancelled are those giving rise to wave-function renormalization. Hoewever, in the end-point region the parton transverse momenta in T_H cannot be *a priori* ignored since, say, for the pion, $\left(\vec{k}_{\perp i} + \vec{k}'_{\perp j}\right)^2 \gg x_i x'_j Q^2$. As a result, the transverse distance between the quark and the antiquark becomes large compared to $1/Q$ and the corresponding gluon line is no more part of the hard-scattering process but should be counted to its soft part. In other words, the hard-gluon exchange mechanism should be replaced by that of Feynman.

The physical basis of the modified convolution scheme (MCS) [19] is to dissect the process in such a way, so that for transverse distances large compared to $1/Q$ (playground of the hard-scattering mechanism) but still small relative to the true confinement

regime – characterized by $1/\Lambda_{QCD}$ – the hadron wave function is modified to exhibit the effect of Sudakov enhancements explicitly up to the transverse scale retained in T_H. Going over to transverse configuration space, the modified wave function reads

$$\hat{\Psi}^{(H)}_{(mod)}(x_i, 1/\tilde{b}_i, Q, \mu_{ren}) = \mathrm{e}^{-S}\hat{\Psi}^{(H)}(x_i, 1/\tilde{b}_i)\ , \tag{10}$$

where the factor

$$\exp\left(-S\right) = \sum_{i=1}^{N}\left[s(x_i, \tilde{b}_i, Q) + \int_{1/\tilde{b}_i}^{\mu_{ren}} \frac{d\overline{\mu}}{\overline{\mu}}\gamma_q(g(\overline{\mu}^2))\right] + \quad x_i \leftrightarrow x'_i \tag{11}$$

comprises gluon corrections in terms of the functions $s(x_i, \tilde{b}_i, Q)$ [19] and accounts for RG-evolution from the IR-scale $1/\tilde{b}_i$ to the renormalization scale μ_{ren} via the quark anomalous dimension (in the axial gauge) $\gamma_q(g(\overline{\mu}^2)) = -\alpha_s/\pi + O(\alpha_s^2)$. The explicit expressions for the Sudakov functions are given in [22]. The Sudakov exponential factor resums contributions from two-particle reducible diagrams (giving rise to double logarithms), whereas two-particle irreducible diagrams (giving rise to single logarithms) are absorbed into the hard scattering amplitude T_H [22]. It can be conceived of as a *finite* renormalization factor to the hadron wave function [15]. The leading double logarithms derive from those momentum regions where soft gluons (all four-momentum components small) and collinear gluons to the external quark lines overlap. These contributions are numerically dominated by the term

$$\exp\left\{-\frac{2C_F}{\beta}\ln\frac{\xi_i Q}{\sqrt{2}\Lambda_{QCD}}\ln\frac{\ln\left(\xi_i Q/\sqrt{2}\Lambda_{QCD}\right)}{\ln\left(1/\tilde{b}_i\Lambda_{QCD}\right)},\right\} \tag{12}$$

where ξ_i is one of the fractions x_i or x'_i, and $\beta = (33-2n_f)/3$ is the first-order term of the Gell-Mann and Low function encountered before. The single logarithm stems from the running coupling constant and the double logarithm contains the exponentiated higher-order corrections – required by RG – rendered finite by the *inherent* IR-cutofff $1/\tilde{b}_i$. This marks the crucial difference between the MCS and previous approaches dealing with *isolated* quarks where such IR-cutoff parameters had to be introduced as *external* regulators.

For small transverse distances (or equivalently, $1/\tilde{b}_i \gg \xi_i Q$), gluonic radiative corrections are treated as being part of T_H and are excluded from the Sudakov form factor. Consequently, for $\xi_i \leq \sqrt{2}/\tilde{b}_i Q$ the Sudakov functions $s(\xi_i, \tilde{b}_i, Q)$ are set equal to zero. On the other hand, as $\tilde{b}_i$ increases e^{-S} decreases, reaching zero at $\tilde{b}_i\Lambda_{QCD} = 1$. In the pion case, there is only one transverse scale, notably, the quark-antiquark separation b, and suppression is automatically accomplished. Indeed, when it happens that one Sudakov function $s(\xi, \tilde{b}_i, Q) = 0$ (or equivalently that the corresponding exponential is set equal to unity) the other (negative) Sudakov function in the exponent, $s(1-\xi, b, Q)$, diverges, thus providing sufficient suppression.

Our approach [24] to the choice of the appropriate IR-cutoff in calculating observables with several transverse momentum scales involved – like nucleon form factors – is to postulate that long wave-length (compared to the typical interquark separations) gluons "see" the nucleon as a *whole*, i.e., in a color-singlet state and cannot resolve its color structure. This is technically implemented by setting $\tilde{b} \equiv \max\{b_1, b_2, b_3\} = \tilde{b}_1 = \tilde{b}_2 = \tilde{b}_3$ ("MAX" prescription). As a result, (i) logarithmic α_s-singularities in the end-point region are *screened* by the exponentially decreasing Sudakov factors, (ii) the form-factor integrands are IR-safe for all possible kinematic configurations and receive perturbative contributions which *saturate*, i.e., which are rather insensitive to distances of order $1/\Lambda_{QCD}$. It was outlined in [24,15] that other choices may lead to uncompensated α_s-singularities. In what follows we present those aspects of the calculation which differ from the standard case. We also present theoretical predictions in comparison with available experimental data. A more detailed level of description is given in [24,15].

All told, the nucleon form factor recast in the transverse configuration space reads

$$\begin{aligned} G_M(Q^2) = \frac{16}{3}\int_0^1 [dx][dx'] \int \frac{d^2 b_1}{(4\pi)^2}\frac{d^2 b_2}{(4\pi)^2} \sum_j \hat{T}_j(x, x', \vec{b}, Q, \mu)\hat{Y}_j(x, x', \vec{b}, \mu) \\ \times \exp\left[-S_j(\xi_i, \tilde{b}_i, Q, \mu)\right], \end{aligned} \tag{13}$$

where the transverse separation vectors between quarks 1 and 3, 2 and 3, and 1 and 2 are defined as follows: $\vec{b}_1 = \vec{b}'_1$, $\vec{b}_2 = \vec{b}'_2$, $\vec{b}_3 = \vec{b}_2 - \vec{b}_1$. The modified wave functions are given by ($\overline{x}_i \equiv 1 - x_i$)

$$\begin{aligned} \hat{Y}_1^{p(n)} &= \frac{1}{\overline{x}_1\overline{x}'_1}\Big\{4(-2)\hat{\Psi}^{\star\prime}_{123}\hat{\Psi}_{123} + 4(-2)\hat{\Psi}^{\star\prime}_{132}\hat{\Psi}_{132} + \hat{\Psi}^{\star\prime}_{231}\hat{\Psi}_{231} + \hat{\Psi}^{\star\prime}_{321}\hat{\Psi}_{321} \\ &\pm\ 2(1)\left[\hat{\Psi}^{\star\prime}_{231}\hat{\Psi}_{132} + \hat{\Psi}^{\star\prime}_{132}\hat{\Psi}_{231} + \hat{\Psi}^{\star\prime}_{321}\hat{\Psi}_{123} + \hat{\Psi}^{\star\prime}_{123}\hat{\Psi}_{321}\right]\Big\}, \end{aligned} \tag{14}$$

$$\begin{aligned} \hat{Y}_2^{p(n)} &= \frac{1(2)}{2\overline{x}_2\overline{x}'_1}\left\{3(0)\hat{\Psi}^{\star\prime}_{132}\hat{\Psi}_{132} \mp \hat{\Psi}^{\star\prime}_{231}\hat{\Psi}_{231} \mp \hat{\Psi}^{\star\prime}_{231}\hat{\Psi}_{132} \mp \hat{\Psi}^{\star\prime}_{132}\hat{\Psi}_{231}\right\} \\ &\mp \frac{1}{\overline{x}_3\overline{x}'_1}\left\{2(1)\left[\hat{\Psi}^{\star\prime}_{321}\hat{\Psi}_{321} + \hat{\Psi}^{\star\prime}_{321}\hat{\Psi}_{123} + \hat{\Psi}^{\star\prime}_{123}\hat{\Psi}_{321}\right] \pm \hat{\Psi}^{\star\prime}_{123}\hat{\Psi}_{123}\right\}, \end{aligned} \tag{15}$$

where the lower signs and the numbers in parentheses refer to the neutron. The diagrams of hard-gluon exchanges in the MCS can be conveniently combined [19] to give

$$\hat{T}_1 = \frac{8}{3}\, C_F\, \alpha_s(t_{11})\alpha_s(t_{12}) K_0\left((\overline{x}_1\overline{x}'_1)^{1/2} Q b_1\right) K_0\left((x_2 x'_2)^{1/2} Q b_2\right), \tag{16}$$

$$\hat{T}_2 = \frac{8}{3}\, C_F\, \alpha_s(t_{21})\alpha_s(t_{22}) K_0\left((x_1 x_1')^{1/2} Q b_1\right) K_0\left((x_2 x_2')^{1/2} Q b_2\right) \ , \qquad (17)$$

where K_0 is the modified Bessel function of order 0 (the Macdonald function). The arguments of the running coupling constant, t_{ji}, are defined as the maximum scale of either the longitudinal momentum $\propto Q$ or the inverse transverse separation $\propto 1/b_i$, appearing in the argument of K_0. They are associated with the virtualities of the exchanged gluons, namely, $t_{11} = \max\left[(\overline{x}_1 \overline{x}_1')^{1/2} Q, 1/b_1\right]$, $t_{21} = \max\left[(x_1 x_1')^{1/2} Q, 1/b_1\right]$, and $t_{12} = t_{22} = \max\left[(x_2 x_2')^{1/2} Q, 1/b_2\right]$. Note that the Fourier transform of the proton wave function reads

$$\hat{\Psi}_{123}(x, \vec{b}, \mu) = \frac{1}{8\sqrt{N_c!}}\, f_N(\mu) \Phi_{123}(x, \mu) \hat{\Omega}_{123}(x, \vec{b}) \ , \qquad (18)$$

where the $k_\perp$-dependent part is modeled by [24]

$$\hat{\Omega}_{123}(x, \vec{b}) = (4\pi)^2 \exp\left[-\frac{1}{4a^2}\left(x_1 x_3 b_1^2 + x_2 x_3 b_2^2 + x_1 x_2 b_3^2\right)\right] \ . \qquad (19)$$

Similar, albeit simplified, expressions are obtained also for the pion form factor [23]. The theoretical predictions are shown in Figs. 2 and 3.

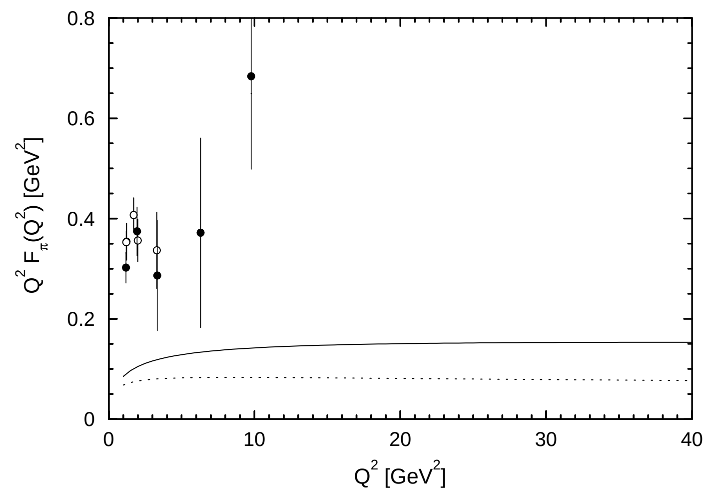

Figure 2: Spacelike pion form factor comprising Sudakov corrections and the intrinsic transverse size of the pion wave function [23] in comparison with experimental data. The curves correspond to the Chernyak-Zhitnisky model [9] (solid line) and the asymptotic wave function (dotted line). The data are from [25].

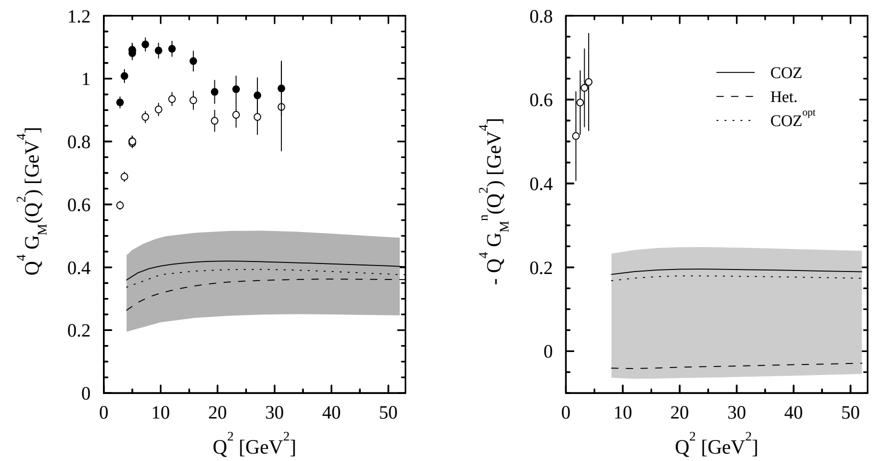

Figure 3: Spacelike magnetic form factors of the nucleon: proton (lhs) and neutron (rhs) ($\Lambda_{QCD} = 180$ MeV). The open circles indicate F_1^p data [26]. The calculated curves have been obtained in [24] with the "MAX" prescription, and including QCD-evolution and the intrinsic $k_\perp$-dependence of the nucleon wave function (the latter normalized to unity). The shaded area contains the predictions derived for the set of nucleon distribution amplitudes determined in [14] via QCD sum rules.

5 Conclusions

The above discussion raises a troubling question. If the self-consistent calculations within the MCS are insufficient to describe the existing data, is all hope lost for the applicability of the pQCD paradigm to exclusive reactions? At one level, no: higher-order corrections, i.e., a K-factor of order 2 – in principle, also computable within the hard-scattering scheme – might bridge the gap to the data. But on a deeper level, the news is sobering. The results may be interpreted as evidence for the failure of the hard-scattering mechanism in exclusive reactions at accessible momentum transfer and this calls for nonperturbative mechanisms.

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